Absolute Mode Number Determination of Frequency Combs

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Abstract— **We demonstrate a method to determine the absolute mode number of a frequency comb when it is used for high precision laser frequency measurement, without the help of wavemeters. Our technique involves changing the repetition rate of the frequency comb in a two-steps process. Guidelines for choosing the correct repetition rates for different laser linewidths are given. As a demonstration, the absolute frequencies of two lasers with different linewidths are measured with our method.**

I. Introduction

The frequency comb technique enables a direct link between microwave and visible optical frequency measurements with one single mode-locked laser, which is a very useful method in high precision laser spectroscopy [1–7]. To measure the frequency of an unknown laser source, the laser frequency f_l, is mixed with the Nth mode of the frequency comb to generate a beat frequency f_b,

$$f_l = Nf_r \pm f_0 \pm f_b, \quad (1)$$

where f_r is the repetition rate and f_0 the offset frequency of the frequency comb, which are locked to a stable frequency standard, e.g., a Cs-standard, such that their values are accurately known. In order to determine f_l from Eq. (1), f_b needs to be measured, and the mode number N needs to be determined. Normally, N is determined by a wavemeter with a resolution better than half of the comb repetition rate. High resolution wavemeters are not only expensive, but also need constant calibration to function properly. L. -S. Ma et al. noted that N can be determined by simply varying the repetition rate of the frequency comb, without the help of a wavemeter [8]. While their method is useful to determine N of a high repetition rate frequency comb, the method becomes less feasible when the laser frequency is measured with a low repetition rate frequency comb.

Here, we demonstrate a new, simpler method that is applicable for both high and low repetition rate frequency combs. Our method is a two-step process. First, a rough estimate of the mode number is obtained by changing the repetition rate of the frequency comb with a small amount. With this knowledge, the repetition rate of the frequency comb is changed one more time to calculate the exact mode number. Guidelines for choosing the correct repetition rates for different laser linewidths are described.

II. Principle

To start, we first determine the signs before f_0 and f_b. The sign of f_b can be determined by observing the relative directional change of f_b with a few Hz change of the repetition rate f_r. With this sign determined, we can determine the sign of f_0 by observing the relative directional change of f_b with small change of f_0. This way all signs can be determined unambiguously in Eq. (1). For simplicity, we assume $f_l = Nf_r + f_0 + f_b$ in this paper. We also lock the offset frequency f_0 to a fixed value in the experiments.

In the experiment, we first obtain a rough estimate of N by changing the repetition rate f_r a small amount such that no mode number change occurs. In this case,

$$\begin{aligned} f_l &= Nf_{r1} + f_0 + f_{b1}, \\ f_l &= Nf_{r2} + f_0 + f_{b2}. \end{aligned} \quad (2)$$

Then N can be estimated as

$$N_{est} = \frac{f_{b2} - f_{b1}}{f_{r1} - f_{r2}}. \quad (3)$$

In order to estimate N as accurately as possible, we need the repetition rate change, $\Delta f_{12} = f_{r1} - f_{r2}$, to be as large as possible without causing a mode number change. For a low repetition rate frequency comb ($f_r \approx 200$ MHz), all visible lasers have a mode number N around 10^6, so the maximum repetition rate change allowed is around 100 Hz. To be on the safe side, we can choose one half of this value, $\Delta f_{12} \leq 50$ Hz.

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From Eq. (3), the uncertainty of N is $\delta N = 1.4\delta f_b / \Delta f_{12}$, where δf_b is determined by the laser fluctuation. If the laser frequency can be determined to within 1 kHz, then δN is approximately 30.

The second step is to improve the accuracy of N. This can be done by changing the repetition rate of the frequency comb by an even larger amount such that the mode number changes. Thus,

$$f_l = Nf_{r1} + f_0 + f_{b1},$$
$$f_l = (N+m)f_{r3} + f_0 + f_{b3}, \tag{4}$$

where f_{b3} is the new beat frequency. The mode number difference m is an integer and can be written as,

$$m = \frac{N\Delta f_{13} + (f_{b1} - f_{b3})}{f_{r3}}. \tag{5}$$

Here, $\Delta f_{13} = f_{r1} - f_{r3}$ is the repetition rate change of the frequency comb in the second step. In order to calculate m, we substitute N with N_{est}, and find that the main uncertainty in m is

$$\delta m = \delta N \Delta f_{13} / f_{r3}. \tag{6}$$

Ideally, δm shall be less than 0.1, which sets an upper limit of $\Delta f_{13} \approx 0.7$ MHz. The lower limit of Δf_{13} is $10\Delta\nu = 10$ kHz. Finally, the mode number N is calculated to be

$$N = \frac{mf_{r3} + (f_{b3} - f_{b1})}{\Delta f_{13}}. \tag{7}$$

With the knowledge of N, the laser frequency f_l from Eq. (1) can be calculated.

The above procedure is robust for a variety of lasers if their frequency fluctuations can be averaged down to less than 10 kHz and if we require that the mode number does not change in the first step of changing repetition rate Δf_{12}. For larger laser fluctuations, a larger Δf_{12} is needed, which means that the mode number will change. In this case, we can monitor the screen of the RF spectrum analyzer to count a small number of mode change m'. The estimated N is then given by

$$N_{est} = \frac{m' f_{r2} + f_{b2} - f_{b1}}{f_{r1} - f_{r2}}. \tag{8}$$

All the remaining calculations are the same. For example, by counting one more mode, m'=1, the measurable laser frequency (beatnote) fluctuation will increase to 20 kHz.

III. DEMONSTRATIONS

To demonstrate the feasibility of our method, we measure the absolute frequencies of two lasers with different linewidths. The frequency comb (MenloSystems FC-8004) has a repetition rate of around 200 MHz, and an offset frequency 20 MHz [9]. Both repetition rate and offset frequency are locked to a Cs frequency standard (Agilent 5071A). The repetition rate is resolution-limited at 1 Hz, measured with a network/spectrum analyzer (HP4195A) with a sweep time of 3 s.

A. *Nd:YAG Laser*

We first measure the absolute frequency of a monolithic isolated end-pumped ring Nd:YAG laser (MISER), used for high resolution indium ion spectroscopy. This Nd:YAG laser has an intrinsically high frequency stability and low amplitude noise. The laser is further locked to a mechanically and thermally isolated high finesse Zerodur reference cavity. The laser linewidth is typically less than 10 Hz [10]. The laser light is sent through a 60 m long polarization-maintaining fiber to beat with the frequency comb. The beatnote uncertainty is measured to be 1.3 kHz at 60 s, which is dominated by fiber noise. The uncertainty of the beatnote can be further reduced by averaging the measured results over a longer period. In order to determine the laser frequency quickly, we choose an averaging time of 60 s as a good compromise. The MISER has a linear drift rate of approximately 1 Hz/s, which has a negligible contribution to the beatnote uncertainty within the measurement time. The measured results are

$$f_{r1} = 201{,}062.036 \text{ kHz}, \quad f_{b1} = 28{,}351.080 \text{ kHz},$$
$$f_{r2} = 201{,}062.031 \text{ kHz}, \quad f_{b2} = 36{,}231.614 \text{ kHz},$$
$$f_{r3} = 201{,}051.956 \text{ kHz}, \quad f_{b3} = 30{,}159.784 \text{ kHz},$$

shown in Fig. 1. The average beat frequencies are calculated with 60 s time averaging. The calculated results are N_{est} = 1576106.8, m = 79.007, and N = 1575884.249, from which we take the integer part 1575884. The laser frequency is determined to be $f_l = Nf_{r1} - f_0 + f_{b1}$ = 316,850,453,890.9 kHz, where the sign before the offset frequency is determined to be minus for this particular example. The result agrees with the previous measurement [11, 12].

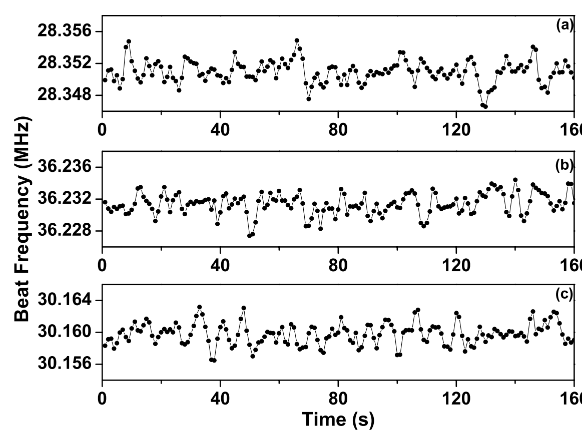

Figure 1. Counted beatnotes between the Nd:YAG laser and the frequency comb under different repetition rates (a) f_{r1} = 201,062.036 kHz; (b) f_{r2} = 201,062.031 kHz; (c) f_{r3} = 201,051.956 kHz. All frequencies are measured using a frequency counter with 1 s gate time.

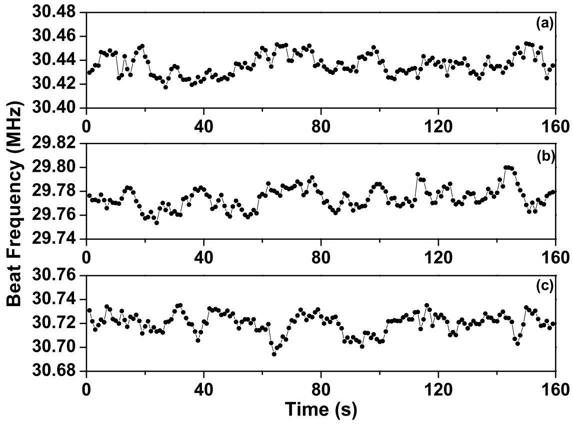

Figure 2. Counted beatnotes between the 922 nm diode laser and the frequency comb under different repetition rates (a) f_{r1} = 201,060.017 kHz; (b) f_{r2} = 201,059.893 kHz; (c) f_{r3} = 200,960.034 kHz. All frequencies are measured using a frequency counter with 1 s gate time.

B. Diode Laser

The second laser measured is a 922 nm diode laser that is locked to an ultra stable, high finesse cavity placed in another temperature stabilized and vibration isolated vacuum chamber. The beatnote uncertainty is measured to be 9.6 kHz at 60 s. This uncertainty borders the zero mode change regime. Hence, we purposely change the repetition rate by Δf_{12} = 124 Hz such that m'=1. The measured results are

$$f_{r1} = 201{,}060.017 \text{ kHz}, \quad f_{b1} = 30{,}436.281 \text{ kHz},$$

$$f_{r2} = 201{,}059.893 \text{ kHz}, \quad f_{b2} = 29{,}774.308 \text{ kHz},$$

$$f_{r3} = 200{,}960.034 \text{ kHz}, \quad f_{b3} = 30{,}719.173 \text{ kHz},$$

and are shown in Fig. 2. The average beat frequencies are again taken with 60 s time averaging. Since we have one mode number change, $N_{est} = (f_{b2} - f_{b1} + f_{r2})/(f_{r1} - f_{r2})$ = 1616112.258, m = 804.058, and N = 1615996.222, from which we take the integer part 1615996. Finally, we obtain $f_l = Nf_{r1} + f_0 + f_{b1}$ = 324,912,233,668 kHz.

IV. CONCLUSIONS

In conclusion, we propose and demonstrate a new method of absolute frequency measurement using only a frequency comb. We use a two-step procedure for determining the mode number of the frequency comb that is closest to the frequency of the laser under measurement. Both steps involve changing the repetition rates of the frequency comb. A guideline for choosing the repetition rates is given. Using this method, we successfully measured the frequencies of two lasers of different linewidths. The measurement results are confirmed by repeating the measurement using the traditional method of combining the frequency comb and a high resolution wavemeter. We note that our method is particularly useful for a low repetition rate frequency comb, and when a high resolution wavemeter is not available.

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